Axiomatizing norms across time and the 'Paradox of the Court'

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Abstract

In normative reasoning one typically refers to intervals of time across which norms are intended to hold, as well as to alternative possibilities representing hypothetical developments of a given scenario. Thus, deontic modalities are naturally intertwined with temporal and metaphysical ones. Furthermore, contemporary debates in philosophy suggest that a proper understanding of fundamental ethical principles, such as the Ought-Implies-Can thesis, requires a simultaneous analysis of these three families of concepts. In the present article we propose a general formal framework which allows for fine-grained multimodal reasoning in the normative domain. We provide an axiomatization for a novel system of propositional logic encoding the way in which possibilities and norms arising from different sources change over intervals of time. The usefulness of our framework is illustrated by analysing an ancient and particularly challenging 'cold case', the Paradox of the Court.

Keywords: Multimodal Reasoning, Norms Across Time, Ought-Implies-Can, Paradox of the Court, Temporal Opportunities.

1 Introduction

One of the most frequently debated principles in moral philosophy, the *Ought-Implies-Can* thesis (OIC), suggests that deontic modalities are essentially connected with other families of modalities. The naive formulation of OIC is the following: if an agent A is obliged to bring about ϕ, then it is possible that A brings about ϕ. But what kind of possibility is here involved? Logical or

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metaphysical possibility is too broad for the message that one wants to convey in terms of OIC. Indeed, what really matters is whether A has a certain *ability* and an *opportunity* of acting in appropriate circumstances to bring about ϕ. While there is a considerable amount of work in the literature focusing on the role played by agents’ abilities in normative reasoning (see, for instance, [6], [7], [15] and [13]), much less has been said on the role played by agents’ opportunities (see, for instance, [28], [21] and the analysis of spatial opportunities in [5]). It seems that both ability and opportunity involve many conceptual dimensions which would be very hard to represent in a single formal framework. In the present article we deal with opportunities from the perspective of *time*: norms are usually expected to apply to specific *temporal intervals* and the possibility of acting in appropriate circumstances to bring about something can be gained or lost during an interval.

For instance, consider the following scenario, adapted from [29]: at 9:00 a student received an order to write a five-page paper by 17:00 as part of an exam. Time passed by and it is now 16:57. The student has not started writing the paper yet. Is there an obligation which applies to the interval between 16:57 and 17:00? At first glance, one would be tempted to give a positive answer, since the student should not be able to justify the outcome of her behaviour by just relying on the flow of time. However, after a closer look at the problem one could argue that the student has actually no obligation to write a five-page paper between 16:57 and 17:00, namely that her original obligation expired. The reason is that the student does not have an opportunity of exercising her ability to write such a long paper in such a short amount of time.

This example shows that an obligation applying to a certain interval of time I is not automatically inherited by all subintervals of I; not even by those subintervals having the same final point as I (e.g., 17:00), namely those subintervals by which I *is finished*, according to the terminology in [1]. Therefore, the fact that at 16:57 it is no longer possible to fulfil the obligation is not a counterexample to OIC; as it is claimed in [30], “obligations that become infeasible at a given time are lost at that time”. This does not mean that no trace of an obligation is left once new conditions make its fulfilment impossible. After 17:00 the student will be blamed and she will not pass the exam, since an obligation applying to a past interval of time, the one between 9:00 and 17:00, will have been violated due to her behaviour.

Furthermore, losing the opportunity to behave in a certain way is often not due to the flow of time *per se*; it is rather due to the fact that new norms become effective over time. A norm applying to an interval I may be overridden by a new norm that is introduced within I when it is not possible to comply with both. Therefore, *normative sources* can play a relevant role in determining when a norm expires. For instance, we can imagine a variation of the scenario above in which the student received a phone call at 9:10 and the person on the phone urgently asked her to go home and assist a family member, thus making impossible for her to write the paper by 17:00. The new obligation clearly takes priority over the old one.

The conclusion we draw from such discussion is that a fine-grained analysis of OIC and other philosophical problems requires making reference not only to deontic and metaphysical modalities, but also —and at least— to temporal ones. In the first part of the present article we will develop a very general formal framework to represent the way in which the three families of modalities at issue are intertwined in normative reasoning. More precisely, we provide an axiomatization for a new logic over a multimodal language making reference to temporal intervals across which norms arising from different sources are expected to hold, as well as to alternative possibilities. Our contribution can be located within the rich and long-lasting tradition of studies on the foundations of *multimodal reasoning with deontic modalities* (some examples are [26], [4], [3] and [24]).

In the second part of the article we will put our framework at work by discussing the *Paradox of the Court*, which is arguably the oldest puzzle for normative reasoning. Such puzzle is described, for instance, in [12]. In Ancient Greece a wealthy young man, Euathlus, became a student of Protagoras, paying him a half of the cost of teaching, and promising to pay him the remaining half on the day he would win his first case. After the end of his education, however, Euathlus changed his mind and decided not to undertake the career of a lawyer. What remained then of the original agreement? Was a payment of the education fee still due? Clever Protagoras thought that there was a way to make sure that the payment would take place. He decided to sue Euathlus, arguing in the following manner ([12], 407):

> Let me tell you, most foolish of youths, that in either event you will have to pay what I am demanding, whether judgment be pronounced for or against you. For if the case goes against you, the money will be due me in accordance with the verdict, because I have won; but if the decision be in your favour, the money will be due me according to our contract, since you will have won a case.

Euathlus, being a clever pupil himself, was not willing to let Protagoras win the argument. He rather saw this as an occasion to make sure that *no* payment would take place. He argued for the opposite conclusion as follows ([12], 409):

> I shall not have to pay what you demand, whether judgment be pronounced for or against me. For if the jurors decide in my favour, according to their verdict nothing will be due you, because I have won; but if they give judgment against me, by the terms of our contract I shall owe you nothing, because I have not won a case.

Prima facie, it seems that both Protagoras and Euathlus are right; despite this, their arguments lead to mutually contradicting conclusions: if Protagoras is right, Euathlus should pay the promised amount of money whether he wins or loses. If Euathlus is right, he is not obliged to pay the promised amount of money whether he wins or loses. This short presentation reveals that the Paradox of the Court is a paradigmatic problem of normative reasoning rooted in the connection between deontic, temporal and metaphysical modalities.

The structure of our article is as follows. In Section 2 we introduce the formal language and the axiomatic basis of our logic **DTM** for reasoning with **D**eontic, **T**emporal and **M**etaphysical modalities; in Section 3 we provide a semantic analysis of **DTM** and a characterization result in terms of a class of intended models, discussing also how the principle OIC can be represented within it. In Section 4, we review some accounts of the Paradox of the Court proposed in the literature. Subsequently, in Section 5, we present our analysis of the paradox in terms of the new logical framework introduced. The article is concluded with an overview of possible applications of **DTM**.

2 Syntax

In this section we describe the multimodal logic **DTM** for reasoning with **D**eontic, **T**emporal and **M**etaphysical modalities. We start by introducing the formal language $\mathcal{L}$ on which the logic is based.

Definition 2.1 (Primitive symbols) *The language $\mathcal{L}$ contains the following primitive symbols:*

- *a countable set of propositional variables VAR, denoted by p, q, r, etc.;*
- *a countable set of normative sources SOU, denoted by s_1, s_2, s_3, etc.;*
- *a countable set of temporal indices IND, denoted by i, j, k, etc.;*
- *the monadic modal operators $\Box_\infty$, $\Box_{[i,j]}$, $\Box_{\Leftarrow i}$ and $\Box_{i\Rightarrow}$, for $i, j \in IND$;*
- *the monadic modal operator L;*
- *the monadic modal operator O^s, for $s \in SOU$;*
- *the binary predicate E taking temporal indices as arguments;*
- *the boolean connectives $\neg$ (negation) and $\to$ (material implication);*
- *round brackets.*

A temporal index can be conceived of as a *non-indexical temporal reference*, namely a particular date or time. For instance, "11 January 2020" or "three days after 5 February 2020" or "Christmas 2020 at 3pm". The intended reading of the primitive symbols in $\mathcal{L}$ will be clarified below, after having specified the set of well-formed formulas.

Definition 2.2 (Well-formed formulas) *The set WFF of* well-formed formulas *of $\mathcal{L}$ is defined by the grammar below (where $p \in VAR$, $i, j \in IND$ and $s \in SOU$), provided that the following two restrictions apply:*

- *in formulas of kind $O^s\phi$, ϕ neither contains occurrences of the predicate E nor of any modal operator different from $\Box_\infty$, $\Box_{[i,j]}$, $\Box_{\Leftarrow i}$ and $\Box_{i\Rightarrow}$;*
- *formulas of kind $\Box_\infty\phi$, $\Box_{[i,j]}\phi$, $\Box_{\Leftarrow i}\phi$ and $\Box_{i\Rightarrow}\phi$ do not include occurrences of operators of kind O^s.*

$$\phi ::= p | E(i,j) | \neg\phi | \phi \to \phi | \Box_\infty\phi | \Box_{[i,j]}\phi | \Box_{\Leftarrow i}\phi | \Box_{i\Rightarrow}\phi | L\phi | O^s\phi$$

Let $ATO = VAR \cup \{E(i,j) : i,j \in IND\}$ be the set of propositional atoms in WFF; elements of ATO will be denoted by a, a', a'', etc. Furthermore, we will denote by WFF^O the subset of WFF including only formulas in which the predicate E never occurs and where the only modal operators (if any) are $\Box_\infty$, $\Box_{[i,j]}$, $\Box_{\Leftarrow i}$ and $\Box_{\Rightarrow i}$ (this set includes precisely the formulas that can be in the scope of an operator O^s, according to the first restriction in Definition 2.2).

A formula of kind $E(i,j)$ means "temporal index i is earlier than temporal index j"; for instance, 11 January 2020 is earlier than 12 January 2020. A formula of kind $\Box_\infty\phi$ means "it is always the case that ϕ"; $\Box_{[i,j]}\phi$ means "throughout the interval between i and j it is always the case that ϕ"; $\Box_{\Leftarrow i}\phi$ means "ϕ is always the case until i"; $\Box_{i\Rightarrow}\phi$ means "ϕ is always the case starting from i"; $O^s\phi$ means "according to normative source s it is obligatory that ϕ"; finally, $L\phi$ means "it is necessarily the case that ϕ".[3] The following formulas can be used as abbreviations, according to usual definitions of boolean and modal operators: $\phi \wedge \psi$, $\phi \vee \psi$, $\phi \equiv \psi$, $\Diamond_\infty\phi$ ("it is sometimes the case that ϕ"), $\Diamond_{[i,j]}\phi$ ("throughout the interval between i and j it is sometimes the case that ϕ"), $\Diamond_{\Leftarrow i}\phi$ ("it is sometimes the case that ϕ until i"), $\Diamond_{i\Rightarrow}\phi$ ("it is sometimes the case that ϕ starting from i"), $P^s\phi$ ("according to normative source s it is permitted that ϕ") and $M\phi$ ("it is possibly the case that ϕ"). For instance, $\Diamond_{[i,j]}\phi := \neg\Box_{[i,j]}\neg\phi$ and $P^s\phi := \neg O^s\neg\phi$. The fact that i is the left index and j the right index in $\Box_{[i,j]}\phi$ does not bear any consequence on whether i is earlier than j; indeed, $\Box_{[j,i]}\phi$ is a well-formed formula as well. The relation earlier/later is rather associated with the predicate E. We will use the expressions $\Box_i$ and $\Diamond_i$ as abbreviations for $\Box_{[i,i]}$ and $\Diamond_{[i,i]}$.

We will now provide a step by step presentation of the axiomatic basis for **DTM**, assuming some familiarity with correspondence theory for modal logic (see, e.g., [27]). First of all, **DTM** is an extension of the classical propositional calculus (**PC**); therefore, we can start developing the axiomatic basis with the following set of principles:[4]

A0+RX All WFF-substitution instances of axioms and rules of **PC**.

Then, we add the following two axioms for the predicate E, which will make the relation of temporal precedence a strict partial order:

A1 $E(i,j) \rightarrow (E(j,k) \rightarrow E(i,k))$;
A2 $\neg E(i,i)$.

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[3] The expression "until" in the reading of $\Box_{\Leftarrow i}\phi$ has an inclusive sense: ϕ is expected to hold also at instant i. For such reason, this operator is more closely related to the "release" operator than to the "until" operator in temporal logics of computation (see, e.g., [9]). Analogously, in a formula of the form $\Box_{\Leftarrow i}\phi$ (respectively, $\Box_{[i,j]}\phi$) the interval considered is inclusive with respect to index i (and index j).

[4] For the sake of brevity, a label of kind An, where n is a natural number, may denote a set of distinct axioms (more precisely, axiom-schemata) and a label of kind Rλ, where λ is an upper case letter, may denote a set of distinct rules. A label of kind An+Rλ denotes the union of all axioms associated with An and all rules associated with Rλ.

A1 and A2 can be used to introduce functions specifying the first and the last temporal index in an interval (analogous functions can be found in a logic for characterizing deadlines in [14]). We use the expression $S(i,j)$ ('i and j are simultaneous') as an abbreviation for $\neg E(i,j) \wedge \neg E(j,i)$. Thus, the predicate S will satisfy the property: $S(i,j) \equiv S(j,i)$.

Definition 2.3 (First and last index in an interval) *Given $i,j \in IND$, let $\alpha[i,j]$ ("the first index in the interval $[i,j]$") be:*

- *i if $E(i,j)$ holds;*
- *j if $E(j,i)$ holds;*
- *both i and j otherwise (that is, if $S(i,j)$ holds).*

Furthermore, let $\omega[i,j]$ ("the last index in the interval $[i,j]$") be:

- *i if $E(j,i)$ holds;*
- *j if $E(i,j)$ holds;*
- *both i and j otherwise (that is, if $S(i,j)$ holds).*

We can now define additional *relations among intervals* in a very simple way, exploiting the functions α and ω, the predicates E and S, and boolean connectives. The labels for these relations are: Ide ("is identical with"), Bef ("is before than"), Mee ("meets"), Ove ("overlaps"), Fin ("is finished by"), Con ("contains") and Sta ("is started by").

Definition 2.4 (Allen-style interval algebra) *Given two intervals $[i,j]$ and $[k,l]$, we have the following fundamental relations among them:*

$$Ide([i,j],[k,l]) := S(\alpha[i,j],\alpha[k,l]) \wedge S(\omega[i,j],\omega[k,l])$$

$$Bef([i,j],[k,l]) := E(\omega[i,j],\alpha[k,l])$$

$$Mee([i,j],[k,l]) := E(\alpha[i,j],\omega[i,j]) \wedge E(\alpha[k,l],\omega[k,l]) \wedge S(\omega[i,j],\alpha[k,l])$$

$$Ove([i,j],[k,l]) := E(\alpha[i,j],\alpha[k,l]) \wedge E(\alpha[k,l],\omega[i,j]) \wedge E(\omega[i,j],\omega[k,l])$$

$$Fin([i,j],[k,l]) := E(\alpha[i,j],\alpha[k,l]) \wedge S(\omega[i,j],\omega[k,l])$$

$$Con([i,j],[k,l]) := E(\alpha[i,j],\alpha[k,l]) \wedge E(\omega[k,l],\omega[i,j])$$

$$Sta([i,j],[k,l]) := S(\alpha[i,j],\alpha[k,l]) \wedge E(\omega[k,l],\omega[i,j])$$

Furthermore, for any interval relation R defined above, one can denote by R^{-1} its converse relation. For instance, $Sta^{-1}([i,j],[k,l]) = Sta([k,l],[i,j])$. Thus, one can represent within **DTM** all thirteen relations described by Allen in [1]. Notice that the following property holds in **DTM**, due to A0+RX, A1 and A2: $Ide^{-1}([i,j],[k,l]) \equiv Ide([i,j],[k,l])$.

Then we move to the analysis of the deductive properties of the operators L and $\Box_\infty$, which are intended to represent metaphysical necessity and temporal necessity (in the sense of truth over any interval of time), respectively. As it is argued in [22], logics of metaphysical necessity should be in the range between

KT and **S5**; we opt for the strongest logic in this range, since it is a very common choice in approaches combining metaphysical and temporal modalities (see, for instance, the approaches to the Paradox of the Court discussed in Section 4).[5] We choose to adopt an **S5** basis for $\Box_\infty$ as well, in order to treat this operator as an interval-based analogue of the notion of Aristotelian necessity defined over linear and transitive temporal structures (see, e.g., [20]). The axiomatic basis is thus extended with the principles below:

A3+RY All axioms and rules of **S5** for L and $\Box_\infty$.

Operators of kind $\Box_{\Leftarrow i}$, $\Box_{i\Rightarrow}$ and $\Box_{[i,j]}$, instead, do not satisfy the axiom T, since they may concern intervals of time to which the current time does not belong. Therefore, we add:

A4+RZ All axioms and rules of **KD45** for operators of kind $\Box_{[i,j]}$, $\Box_{\Leftarrow i}$ and $\Box_{i\Rightarrow}$.

Now, we need to ensure that modal operators can capture all intended properties of temporal intervals. One of these properties is that there is only one point in an interval of kind $[i,i]$. Therefore, we add the following principle:

A5 $\Diamond_i \phi \to \Box_i \phi$.

Then, we need to ensure that $[i,j]$ and $[j,i]$ are two ways of looking at the same interval, and this is a consequence of adding the principle below:

A6 $\Box_{[i,j]}\phi \equiv \Box_{[j,i]}\phi$.

After this, we encode the temporal algebra over the set of intervals via the following axioms:

A7 $Ide([i,j],[k,l]) \to (\Box_{[i,j]}\phi \to \Box_{[k,l]}\phi)$;
A8 $Ove([i,j],[k,l]) \to (\Box_{[i,j]}\phi \to \Box_{\alpha[k,l]}\phi) \wedge (\Box_{[k,l]}\psi \to \Box_{\omega[i,j]}\psi)$;
A9 $Mee([i,j],[k,l]) \to (\Diamond_{\omega[i,j]}\phi \equiv \Diamond_{\alpha[k,l]}\phi)$;
A10 $Con([i,j],[k,l]) \to (\Box_{[i,j]}\phi \to \Box_{[k,l]}\phi)$;
A11 $Sta([i,j],[k,l]) \to ((\Box_{[i,j]}\phi \to \Box_{[k,l]}\phi) \wedge (\Box_{[k,l]}\psi \to \Diamond_{\alpha[i,j]}\psi))$;
A12 $Fin([i,j],[k,l]) \to ((\Box_{[i,j]}\phi \to \Box_{[k,l]}\phi) \wedge (\Box_{[k,l]}\psi \to \Diamond_{\omega[i,j]}\psi))$.

For instance, A7, together with the fact that **DTM** is closed under the schema $Ide([i,j],[k,l]) \equiv Ide^{-1}([i,j],[k,l])$, says that two identical temporal intervals are indistinguishable with respect to the truth of formulas in states they con-

[5] Names of systems and axioms of modal logic adhere to the presentation in [8].

tain: if something is always the case between February 14 (i) and December 25 (j) of a particular year, then it is always the case between Valentine's Day (k) and Christmas (l) of that year, and vice versa.

The next step is making sure that operators of kind $\Box_\infty$, $\Box_{[i,j]}$, $\Box_{\Leftarrow i}$ and $\Box_{i\Rightarrow}$ are related in an appropriate way. This requires also principles combining them with the predicate E. Thus, we add: [6]

A13 $\Box_\infty\phi \equiv (\Box_{\Leftarrow i}\phi \wedge \Box_{i\Rightarrow}\phi)$;
A14 $(\Box_{\Leftarrow i}\phi \wedge \neg E(i,j) \wedge \neg E(i,k)) \rightarrow \Box_{[j,k]}\phi$;
A15 $(\Box_{i\Rightarrow}\phi \wedge \neg E(j,i) \wedge \neg E(k,i)) \rightarrow \Box_{[j,k]}\phi$.

Furthermore, let int and int' be arbitrary intervals, that is, strings of any of the following kinds: either ∞ or $[i,j]$ or $\Leftarrow i$ or $i \Rightarrow$; we add to the axiomatic basis the following bridge-axioms connecting different modalities:

A16 $\Box_{int}\phi \equiv \Box_{int'}\Box_{int}\phi$;
A17 $E(i,j) \rightarrow L\Box_{int}E(i,j)$;
A18 $\Box_{int}L\phi \equiv L\Box_{int}\phi$.

Finally, given that **DTM** is intended to capture minimal relations among deontic, temporal and metaphysical modalities, and that there are several arguments in the literature supporting the idea that deontic modalities are hyperintensional (see, e.g., [10] and [11]), we do not impose any deductive property on operators of kind O^s, except for the following bridge-axiom:

A19 $O^s\phi \rightarrow M\phi$.

In the end, we get the following definition:

Definition 2.5 (Axiomatic basis) *The axiomatic basis for the logic* **DTM** *corresponds with the list of axioms* A0-A19 *and the rules* RX, RY *and* RZ.

The principle A19 can be taken as the formal analogue of the naive formulation of the Ought-Implies-Can thesis. However, in the present framework we can provide a more-fine grained analysis of OIC, taking into account temporal intervals. For instance, as the example from [29] that we discussed in Section 1 shows, one could say that if it is obligatory that ϕ occurs within a certain interval $[i,j]$ and we are at a point k within $[i,j]$, then there is a possible development of the world in which ϕ occurs within the interval $[k,j]$. This is a way of explicitly taking into account the temporal opportunity of bringing about ϕ between k and the time in which the obligation was originally supposed to

[6] Axiom A13 can be also taken as a definition. This allows one to remove $\Box_\infty$ from the set of primitive symbols, provided that some changes in the axiomatic basis are made; for instance, A17 becomes $E(i,j) \rightarrow L(E(i,j) \wedge \Box_{int}E(i,j))$.

expire (j). Thus, we can formulate this version of OIC as follows:

$$(O^s\Diamond_{[i,j]}\phi \wedge E(i,k) \wedge \neg E(j,k)) \rightarrow (O^s\Diamond_{[k,j]}\phi \rightarrow M\Diamond_{[k,j]}\phi)$$

One can use this schema to distinguish between those obligations that are still in effect at a time and those that are not: even if an obligation concerning a temporal interval I is assumed for deductive reasoning, this does not entail that that obligation is effective when we reason about some point within I.

3 Semantics

In this section we describe the *intended* class of frames and models to interpret the logic **DTM**. Let $\mathbb{R}$ be the set of all relations R_{int} such that *int* is an interval. In analogy with what we did in the syntactic part, we will use R_i as a shorthand for $R_{[i,i]}$.

Definition 3.1 (Frames) *The language $\mathcal{L}$ is interpreted on relational frames of kind $\mathfrak{F} = \langle W, \mathbb{R}, A, < \rangle$ where:*

- *W is a set of* states *denoted by w, v, u, etc.;*
- *for any $R_{int} \in \mathbb{R}$, $R_{int} \subseteq W \times W$ is a* "temporal inspection" *relation;*
- *$A \subseteq W \times W$ is a* "metaphysical inspection" *relation;*
- *$< \subseteq W \times IND \times IND$ is a* "temporal precedence" *relation.*

For any $w \in W$, we have $R_{int}(w) = \{v : wR_{int}v\}$ and this can be called the R_{int}*-sphere* of w. An analogous notation can be used with reference to the other relations in a frame.

Definition 3.2 (Models) *A model over a frame $\mathfrak{F}$ is a structure of kind $\mathfrak{M} = \langle \mathfrak{F}, V, N \rangle$ such that:*

- *$V : ATO \longrightarrow \wp(W)$ is a* valuation function*;*
- *for any $s \in SOU$, $N^s : W \longrightarrow WFF^O$ is a* norm assignment *with respect to source s.*

For any $s \in SOU$ and $w \in W$, $N^s(w) \subseteq WFF^O$ is the N^s*-sphere* of w. We want to highlight the fact that the N^s-sphere of a state is model-dependent, whereas any R_{int}-sphere of a state is frame-dependent. Furthermore, in a frame the $<$-sphere of a state w can be different (in general) from the $<$-sphere of a state v; therefore, we will speak of the Allen relation between two intervals $[i, j]$ and $[k, l]$ *as seen from* a state w.

Definition 3.3 (Truth-conditions) *The truth of a formula with reference to a state w in a model $\mathfrak{M}$ is defined below, where $a \in ATO$ and $s \in SOU$:*

- *$\mathfrak{M}, w \models a$ iff $w \in V(a)$;*
- *$\mathfrak{M}, w \models \neg\phi$ iff $\mathfrak{M}, w \not\models \phi$;*
- *$\mathfrak{M}, w \models \phi \rightarrow \psi$ iff either $\mathfrak{M}, w \not\models \phi$ or $\mathfrak{M}, w \models \psi$;*
- *$\mathfrak{M}, w \models \Box_{int}\phi$ iff $\mathfrak{M}, v \models \phi$ for all $v \in R_{int}(w)$;*
- *$\mathfrak{M}, w \models O^s\phi$ iff $\phi \in N^s(w)$;*

- $\mathfrak{M}, w \models L\phi$ *iff* $\mathfrak{M}, v \models \phi$ *for all* $v \in A(w)$.

A formula ϕ is valid in a model $\mathfrak{M}$ iff ϕ is true at all states in the domain of $\mathfrak{M}$; ϕ is valid in a frame $\mathfrak{F}$ iff it is valid in all models over $\mathfrak{F}$. Validity in a class of frames/models is validity in all frames/models of the class.

We will denote by $R \circ R'$ the *composition* or two relations R and R'.

Definition 3.4 (Intended frames) *The class of intended frames for* **DTM**, *denoted by* C_f*, is the class of all frames such that:*

Πa *for every* $i, j \in IND$, $R_{[i,j]}$, $R_{\Leftarrow i}$ *and* $R_{i\Rightarrow}$ *are serial, transitive and euclidean relations;*
Πb A *and* R_∞ *are equivalence relations;*
Πc $<$ *is a strict partial order;*
Πd *for any* $i \in IND$, *if* $v, u \in R_i(w)$, *then* $u = v$;
Πe *for any* $i, j \in IND$, $R_{[i,j]} = R_{[j,i]}$;
Πf *for any* $i, j, k, l \in IND$, *the following properties are associated with the Allen relation among* $[i, j]$ *and* $[k, l]$ *as seen from* w:
- *if* $[i, j]$ *and* $[k, l]$ *are identical, then* $R_{[i,j]}(w) = R_{[k,l]}(w)$
- *if* $[i, j]$ *overlaps with* $[k, l]$, *then* $R_{\omega[i,j]}(w) \subseteq R_{[k,l]}(w)$ *and* $R_{\alpha[k,l]}(w) \subseteq R_{[i,j]}(w)$
- *if* $[i, j]$ *meets* $[k, l]$, *then* $R_{\omega[i,j]}(w) = R_{\alpha[k,l]}(w)$
- *if* $[i, j]$ *is finished by* $[k, l]$, *then* $R_{[k,l]}(w) \subseteq R_{[i,j]}(w)$ *and* $R_{\omega[i,j]}(w) \cap R_{[k,l]}(w) \neq \emptyset$
- *if* $[i, j]$ *is started by* $[k, l]$, *then* $R_{[k,l]}(w) \subseteq R_{[i,j]}(w)$ *and* $R_{\alpha[i,j]}(w) \cap R_{[k,l]}(w) \neq \emptyset$
- *if* $[i, j]$ *contains* $[k, l]$, *then* $R_{[k,l]}(w) \subseteq R_{[i,j]}(w)$;

Πg *for any* $i \in IND$, $R_\infty = R_{\Leftarrow i} \cup R_{i\Rightarrow}$;
Πh *for any* $i, j, k \in IND$, *if* $(i, j), (i, k) \notin< (w)$ *then* $R_{[j,k]}(w) \subseteq R_{\Leftarrow i}(w)$;
Πi *for any* $i, j, k \in IND$, *if* $(j, i), (k, i) \notin< (w)$ *then* $R_{[j,k]}(w) \subseteq R_{i\Rightarrow}(w)$;
Πj $R_{int} \circ R_{int'} = R_{int}$;
Πk $< (w) = < (v)$ *whenever* $v \in (A \circ R_{int})(w)$ *(for some* R_{int}*)*;
Πl $R_{int} \circ A = A \circ R_{int}$.

Definition 3.5 (Intended models) *The class of intended models for* **DTM**, *denoted by* C_m*, is the class of all models over frames in* C_f *such that:*[7]

Πx *for any* $i, j \in IND$, $w \in V(E(i, j))$ *iff* $(i, j) \in< (w)$;
Πy *for any* $s \in SOU$, $\phi \in N^s(w)$ *only if there is* $v \in A(w)$ *s.t.* $\mathfrak{M}, v \models \phi$.

[7] Due to the syntactic restrictions specified in Definition 2.2, the N^s-sphere of a state w (for every $s \in SOU$) includes only formulas where deontic operators never occur, and the truth of such formulas at a state v of a model $\mathfrak{M}$ can be established without any reference to the N^s-sphere of w. This ensures that property Πy is not defined in a circular way.

Before moving to the semantic characterization of **DTM**, we would like to briefly comment on some philosophical points. In the present logical framework the interaction between truth of formulas, states in a model and temporal indices captures the relation between the *flow of time* and *change* along the following lines. First, we can say that a maximal and **DTM**-consistent set of formulas in WFF constitutes a *configuration of the world*. Second, states in a model can be said to be *pictures of a configuration of the world*, since each of them is associated with a maximal and **DTM**-consistent set of formulas (though, this is not in general a one-to-one correspondence, since there are models in which two states are associated with the same configuration). Third, according to property Πd, exactly one picture of a configuration of the world is associated with each temporal index. However, the same picture can be associated with successive temporal indices, since this depends on the level of temporal granularity of a representation (for instance, one might have a new picture of a configuration of the world every second day, every second month, etc.).

Proposition 3.6 *The system* **DTM** *is sound w.r.t. the class* C_m.

Proof. An induction on the length of derivations. First we consider axioms: in the case of A0-A6, A13, A16, A18 and A19 the proof is a standard procedure in propositional (multimodal) reasoning; we here illustrate the other cases.

Consider A7. Assume that we have a model $\mathfrak{M}$ in C_m and a state w in its domain s.t. $\mathfrak{M}, w \vDash Ide([i,j],[k,l])$ and $\mathfrak{M}, w \vDash \Box_{[i,j]}\phi$ but $\mathfrak{M}, w \nvDash \Box_{[k,l]}\phi$. Thus, (I) for all states $v \in R_{[i,j]}(w)$, we have $\mathfrak{M}, v \vDash \phi$ and (II) there is a state $u \in R_{[k,l]}(w)$ s.t. $\mathfrak{M}, u \nvDash \phi$. However, the intervals $[i,j]$ and $[k,l]$ are identical as seen from w and so, due to property Πf, $R_{[i,j]}(w) = R_{[k,l]}(w)$ and a contradiction can be obtained.

Consider A8. Assume that $\mathfrak{M}, w \vDash Ove([i,j],[k,l])$ but $\mathfrak{M}, w \nvDash (\Box_{[k,l]}\phi \rightarrow \Box_{\omega[i,j]}\phi) \wedge (\Box_{[i,j]}\psi \rightarrow \Box_{\alpha[k,l]}\psi)$, for some $\phi, \psi \in WFF$. Let $\mathfrak{M}, w \nvDash \Box_{[k,l]}\phi \rightarrow \Box_{w[i,j]}\phi$. We know that (I) $[i,j]$ overlaps $[k,l]$ as seen from w, and (II) for all $v \in R_{[k,l]}(w)$ we have $\mathfrak{M}, v \vDash \phi$. Due to properties Πa and Πf, there is some state $u \in R_{\omega[i,j]}$, and $u \in R_{[k,l]}(w)$. Therefore, $\mathfrak{M}, u \vDash \phi$. Furthermore, due to Πd, u is the only state in $R_{\omega[i,j]}(w)$, so $\mathfrak{M}, w \vDash \Box_{\omega[i,j]}\phi$: contradiction. The argument for $\mathfrak{M} \nvDash \Box_{[i,j]}\psi \rightarrow \Box_{\alpha[k,l]}\psi$ is analogous.

Consider A9. Assume that $\mathfrak{M}, w \vDash Mee([i,j],[k,l])$ but $\mathfrak{M}, w \nvDash \Diamond_{\omega[i,j]}\phi \equiv \Diamond_{\alpha[k,l]}\phi$ for some $\phi \in WFF$. We can focus, without loss of generality, on the case in which $\mathfrak{M}, w \vDash \Diamond_{\omega[i,j]}\phi$ and $\mathfrak{M}, w \nvDash \Diamond_{\alpha[k,l]}\phi$. Thus, (I) there is $v \in R_{\omega[i,j]}(w)$ s.t. $\mathfrak{M}, v \vDash \phi$, and (II) for all $u \in R_{\alpha[k,l]}(w)$ we have $\mathfrak{M}, u \nvDash \phi$. However, since $[i,j]$ meets $[k,l]$ as seen from w, then $R_{\omega[i,j]}(w) = R_{\alpha[k,l]}(w)$ and we get a contradiction.

Consider A10. Assume $\mathfrak{M}, w \vDash Con([i,j],[k,l])$ and $\mathfrak{M}, w \nvDash \Box_{[i,j]}\phi \rightarrow \Box_{[k,l]}\phi$ for some $\phi \in WFF$. From this one can infer that (I) for all $v \in R_{[i,j]}(w)$, we have $\mathfrak{M}, v \vDash \phi$, and (II) for some $u \in R_{[k,l]}(w)$, we have $\mathfrak{M}, u \nvDash \phi$. However, since $[i,j]$ contains $[k,l]$ as seen from w, then $R_{[k,l]}(w) \subseteq R_{[i,j]}$ and we get a contradiction.

Consider A11. Assume that $\mathfrak{M}, w \vDash Sta([i,j],[k,l])$ and $\mathfrak{M}, w \nvDash (\Box_{[i,j]}\phi \rightarrow \Box_{[k,l]}\phi) \wedge (\Box_{[k,l]}\psi \rightarrow \Diamond_{\alpha[i,j]}\psi)$, for some $\phi, \psi \in WFF$. Let $\mathfrak{M}, w \nvDash \Box_{[i,j]}\phi \rightarrow \Box_{[k,l]}\phi$. Therefore, (I) for all $v \in R_{[i,j]}(w)$, we have $\mathfrak{M}, v \vDash \phi$, and (II) there is some $u \in R_{[k,l]}(w)$ s.t. $\mathfrak{M}, u \nvDash \phi$. However, since $[i,j]$ is started by $[k,l]$ as seen from w, then $R_{[k,l]}(w) \subseteq R_{[i,j]}(w)$ and we get a contradiction. Let $\mathfrak{M}, w \nvDash \Box_{[k,l]}\psi \rightarrow \Diamond_{\alpha[i,j]}\psi$. Therefore, (I) for all $v \in R_{[k,l]}(w)$ we have $\mathfrak{M}, v \vDash \psi$, and (II) there is no $u \in R_{\alpha[i,j]}(w)$ s.t. $\mathfrak{M}, u \vDash \psi$. However, since $[i,j]$ is started by $[k,l]$ as seen from w, then $R_{\alpha[i,j]}(w) \cap R_{[k,l]}(w) \neq \emptyset$ and this leads to a contradiction. The argument for A12 is analogous.

Consider A14. Assume $\mathfrak{M}, w \vDash (\Box_{\Leftarrow i}\phi \wedge \neg E(i,j) \wedge \neg E(i,k))$ and $\mathfrak{M}, w \nvDash \Box_{[j,k]}\phi$ for some $\phi \in WFF$. Then, for all $v \in R_{\Leftarrow i}(w)$ we have $\mathfrak{M}, v \vDash \phi$; furthermore, $(i,j),(i,k) \notin < (w)$. Due to property Πh we have that $R_{[j,k]}(w) \subseteq R_{\Leftarrow i}(w)$ and we get a contradiction. The argument for A15 is analogous.

Consider A17. Assume $\mathfrak{M}, w \vDash E(i,j)$ but $\mathfrak{M}, w \nvDash L\Box_{int}E(i,j)$ for some interval int. Then, $(i,j) \in < (w)$ and there is $v \in (A \circ R_{int})(w)$ s.t. $\mathfrak{M}, v \nvDash E(i,j)$. However, by property Πk, $< (w) = < (v)$, and we get a contradiction.

The fact that rules RX, RY and RZ preserve validity in every model in C_m is straightforward. $\Box$

Proposition 3.7 *The system* **DTM** *is complete w.r.t. the class* C_m.

Proof. The canonical frame $\mathfrak{F}$ for **DTM** can be built following the usual steps for systems of modal logic, with the only difference that for every maximal consistent set of formulas w, every $i,j \in IND$ and every $s \in SOU$ we have:

- $< (w) = \{(i,j) : E(i,j) \in w\}$;
- $N^s(w) \subseteq WFF^O$.

The canonical model $\mathfrak{M}$ over $\mathfrak{F}$ is such that, for every maximal consistent set of formulas w, and every $a \in ATO$, we have:

- $V(a) = \{w : a \in w\}$.

We now illustrate that the canonical model belongs to the class C_m.

The proof that $\mathfrak{M}$ satisfies properties Πa–Πe, Πg and Πj–Πl relies on standard arguments in completeness results for modal propositional logic (in the case of Πc only basic propositional reasoning with $A1$ and $A2$ is needed). We will analyse how the remaining properties of models in C_m (and of the underlying frames) are satisfied.

In the case of Πf we illustrate one example. Assume that the interval $[i,j]$ contains the interval $[k,l]$ as seen from a state w but $R_{[k,l]}(w) \nsubseteq R_{[i,j]}(w)$. Then there is a state $v \in W$ s.t. $v \in R_{[k,l]}(w)$ and $v \notin R_{[i,j]}(w)$. From this one can infer that $\{\phi : \Box_{[k,l]}\phi \in w\} \subseteq v$ and that there is some $\psi \in WFF$ s.t. $\Box_{[i,j]}\psi \in w$ and $\psi \notin v$. However, since $[i,j]$ contains $[k,l]$ as seen from w, then $(\alpha(i,j),\alpha(k,l)),(\omega(k,l),\omega(i,j)) \in <(w)$ and this entails $Con([i,j],[k,l]) \in w$. Furthermore, since w is closed under A10, then $\Box_{[k,l]}\psi \in w$, whence $\psi \in v$: contradiction.

In the case of Πh, assume that $(i,j),(i,k) \notin <(w)$ but $R_{[j,k]}(w) \nsubseteq R_{\Leftarrow i}(w)$. Then there is some state $v \in R_{[j,k]}(w)$ such that $v \notin R_{\Leftarrow i}(w)$. From this one can infer that $\{\phi : \Box_{[j,k]}\phi \in w\} \subseteq v$ and that there is ψ s.t. $\Box_{\Leftarrow i}\psi \in w$ and $\psi \notin v$. However, since $\neg E(i,j) \wedge \neg E(i,k) \in w$ and w contains all instances of A14, then $\Box_{[j,k]}\psi \in w$ and $\psi \in v$: contradiction.

Property Πx is satisfied due to the definition of $\mathfrak{F}$ and $\mathfrak{M}$. Finally, consider property Πy. Suppose there are $s \in SOU$ and $\phi \in WFF^O$ s.t. $\phi \in N^s(w)$, and that for no $v \in A(w)$ we have $\mathfrak{M}, v \vDash \phi$. Then, for any such v we have $\phi \notin v$, whence $M\phi \notin w$; however $\mathfrak{M}, w \vDash O^s\phi$, so $O^s\phi \in w$ and we get a contradiction in the light of A19.

□

4 The Paradox of the Court: Proposed Solutions

Many formal accounts of the Paradox of the Court, also known as *Protagoras v. Euathlus*, have been provided in the literature. In [16] Lenzen offers an analysis within a so-called *base logic*. As [18] neatly puts it, this logic "is defined by the axioms of the classical sentence calculus, axioms of necessity operator of the modal system **S5**, and axioms of identity predicate. The only inference rule is Modus Ponens." This approach is improved by Åqvist in [2], in terms of temporal deontic logic and the definition of several interesting notions, such as (in)validity as applied to an agreement, (in)correctness as applied to a verdict, or the import of an agreement (what follows from it). Both Lenzen and Åqvist aim at *solving the paradox* by deriving in their formal systems a way of getting paid the established fee.

Smullyan in [25] proposes an informal solution, which is suggested to him by "a lawyer" and goes as follows:

> The court should award the case to the student —the student shouldn't have to pay, since he hasn't yet won his first case. After the termination of the case, *then* the student owes money to Protagoras, so Protagoras should then turn around and sue the student a second time. This time, the court should award the case to Protagoras, since the student has now won his first case.

Rescher [23] agrees with Smullyan that the two-trials solution appears to have the strongest claim.

Łukowski summarizes and criticizes various solutions to the Paradox of the Court, proposing his own solution, in [17] and [18]. He objects that many accounts available in the literature, including the logical reconstructions by Lenzen and Åqvist, substitute a *legal pseudo-problem* (getting paid the established fee) for the ancient *logical dilemma*. The original logical problem consists in the contradiction resulting from putting the conclusions of two arguments that are *equally plausible* —one formulated by Protagoras, another one by Euathlus. The legal pseudo-problem can be managed only within the second case mentioned by Smullyan. But Łukowski notes that no second case is mentioned in the original paradox, and that the real problem clearly pertains to the first (and only) case.

We partly agree with Łukowski's criticism. However, we are also afraid that his own solution is open to similar objections as those he raises. He argues as follows:

> Let us use two expressions: 'pay the agreed *fee*' and 'pay the court ordered *damages*' rather than 'pay for the education'. It is easy to see that such a simple operation eliminates the unwanted contradiction. If Euathlus wins the court case, he must pay the agreed fee, even though he does not pay the damages. If Protagoras is the winner, the situation will be quite the opposite.

The conclusion of the argument is that Euathlus has to pay the fee in both cases, and the contradiction thus vanishes. The problem with this account is that, just as the *second case* has not been mentioned in the original paradox, *damages* have not been mentioned either. Because of this, while Łukowski's solution appears to be on the right track, it is not entirely satisfactory. Lenzen and Åqvist importantly show that the temporal aspect is important to understand the paradox; Łukowski shows that an ambiguity of a certain kind is lurking behind the paradox. We will combine these intuitions by proposing a novel account of the paradox within the formal framework introduced in this article.

5 Representing the Paradox of the Court in Our Framework

Where is the ambiguity at the basis of the Paradox of the Court to be located, exactly? In the original formulation, there is no ambiguity in what should be paid, that is, there is no distinction between the *fee* versus the court ordered *damages*. Damages are not mentioned at all, it is all about the money for education (the fee). The ambiguity is rather rooted in *different sources of norms.*[8] Recall this passage from [12] (italics added): "For if the case goes against you, *the money will be due me in accordance with the verdict*, because I have won; but if the decision be in your favour, *the money will be due me according to our contract*, since you will have won a case." We thereby suggest that the difference is between being obliged (or not being obliged) to pay the fee in accordance with the verdict versus to pay the fee in accordance with the agreement.

Euathlus might not be obliged to pay the fees on the basis of the court decision (if he wins the court case); but it does not follow that there is no other obligation - on the basis of the agreement - to pay Protagoras for his teaching. And the other way around, Euathlus might not be obliged to pay the fee on the basis of the agreement (if he loses the court case), but it does not follow that there is no other obligation to pay Protagoras for his teaching.

Let us now capture these intuitions in the proposed formal language. To begin with, we need to distinguish five different things surrounding the paradox:

- the agreement between Protagoras and Euathlus;

[8] Given that the promise or the agreement in question is sufficient for generating the obligation; cf. [19] for a more complex procedural treatment of promises.

- the argument by Protagoras leading to the conclusion that Euathlus is obliged to pay;
- the argument by Euathlus leading to the conclusion that Euathlus is not obliged to pay;
- the scenario that (apparently) took place;
- the scenario that should have taken place.

Let p be the proposition that Euathlus wins the first court case and let q be the proposition that Euathlus pays the fee. In addition, let O^a represent an obligation on the basis of the agreement between Protagoras and Euathlus and O^c represent an obligation on the basis of the court decision. Let i be the date when the education terminated.

The Agreement. In the proposed language, the agreement can be analysed by saying that in all possible courses of events, if there is a case that Euathlus wins at a time j after i, then Euathlus is obliged to pay the fee starting from j. Thus, we have the following schema, for all $j \in IND$:

$$E(i, j) \to L(\Diamond_j p \to O^a \Diamond_{j\Rightarrow} q)$$

The Argument Formulated by Protagoras. Protagoras breaks down the possible outcomes of his trial against the former scholar into two options:

- Euathlus wins the case.
- Euathlus does not win the case.

Let r stand for the proposition that Euathlus wins the case and let k be the date of the court decision. Thus, at k, either r holds or $\neg r$ holds. If r holds, Protagoras argues, $O^a \Diamond_{k\Rightarrow} q$ holds too, since the conditions of the agreement are satisfied. If, on the other hand, $\neg r$ holds, $O^c \Diamond_{k\Rightarrow} q$ holds too, because the court decided in favour of Protagoras. We can immediately identify one mistake in the argument formulated by Protagoras: If Euathlus does not win, it does not follow that the court ordered Euathlus to pay (i.e., that Protagoras wins), as it will be clear in the description of the scenario that (apparently) took place.

The Argument Formulated by Euathlus. Euathlus too, breaks the possible outcomes of the trial into two options. If r holds, Euathlus argues, the jurors decided in his favour, so he does not have to pay; i.e., $\neg O^c \Diamond_{k\Rightarrow} q$. If, on the other hand, $\neg r$ holds, then he has not won any case yet, so he is not obliged to pay according to the agreement; i.e., $\neg O^a \Diamond_{k\Rightarrow} q$. Here we can again spot a mistake: it does not follow from not being obliged to pay on the basis of one normative source that there is no other normative source that obliges one to pay. In other words, if r holds, then really $\neg O^c \Diamond_{k\Rightarrow} q$ holds, but also $O^a \Diamond_{k\Rightarrow} q$ holds. And if $\neg r$ holds, then really $\neg O^a \Diamond_{k\Rightarrow} q$ holds, but it can be that $O^c \Diamond_{k\Rightarrow} q$ holds too (i.e., that Protagoras wins).

The Actual Scenario. In the actual scenario (as presented by Gellius; [12], 409), "the jurors, thinking that the plea on both sides was uncertain and insoluble, for fear that their decision, for whichever side it was rendered, might annul itself, left the matter undecided and postponed the case to a distant day." At k, the conditions that there is some j such that $E(i, j)$ and $\Diamond_j p$ are not satisfied, because Euathlus has not won the given case (no one has), nor has he won any other case yet. He is thus not obliged to pay the fees —neither on the basis of the court decision nor on the basis of the agreement. We thus have $\neg O^c \Diamond_{k\Rightarrow} q$, but also $\neg O^a \Diamond_{k\Rightarrow} q$.

The (Legally) Ideal Scenario. In the legally ideal scenario described above by Smullyan, Euathlus wins the first case because, till the court decision, he has not won any case, and thus is not obliged to pay. However, after this victory he can be sued another time and be obliged to pay. In other words, until k, the conditions that there is some j such that $E(i, j)$ and $\Diamond_j p$ are not satisfied. However, after k, those conditions can be satisfied.

6 Final Remarks

This paper proposed a fine-grained formal framework for normative reasoning that combines deontic, temporal, and metaphysical modalities. The main motivation for this choice is the fact that in normative reasoning, as well as in fundamental ethical principles, such as the Ought-Implies-Can principle (OIC), modalities of these three kinds are intertwined. Admittedly, OIC is a rather complex principle; in the present paper, we have dealt with only one aspect of OIC – the agent's need of a temporal opportunity to fulfil an obligation.

Furthermore, we illustrated how the framework works in formalizing a troublesome ancient paradox, the Paradox of the Court. We proposed a new account of the paradox, which takes some inspiration from existing accounts and highlights the following aspects: the temporal dimension; the presence of ambiguity; the hidden mistakes of the two arguments leading to contradictory results.

There are several interesting directions for future research. One direction is exploring how the families of modalities in question are intertwined in other issues related to ethics and morality; for instance, in debates around moral or legal *responsibility*, where reference to alternative possibilities and temporal opportunities is fundamental to evaluating the behaviour of a normative party. Exploring these issues in a formal framework that is very simple (being based on a propositional multimodal language), but rich enough to specify how obligations and possibilities are lost or gained across intervals of time, could be very useful. Yet another direction would be developing axiomatic extensions of the minimal logic **DTM** presented here in order to encode further principles of normative reasoning. For instance, in order to account for an agent's freedom, one could extend the formal representation of OIC with a condition making reference to the metaphysical possibility of violating obligations.

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